Profit Sharing: A Contracting Solution to Harness the Wisdom of the Crowd

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March 2017

Wisdom of the crowd

Wisdom of the crowd (Surowiecki (2005))

- the collective opinion of a group of individuals
- often found to be better than the judgment of a single expert

Why do we care about the wisdom of the crowd?

- individual judgments often contain idiosyncratic noises
  - ▷ averaging tends to cancel out these noises (law of large numbers)
- rooted in classic economic thoughts
  - ▷ Hayek (1944, 1945), Hellwig (1980), Diamond and Verrecchia (1981)
- useful for modern settings (e.g. earnings forecast, crowdfunding)
  - ▷ Da and Huang (2015), Brown and Davies (2015)

This paper (assuming the existence of the wisdom of the crowd effect):

- how to best harness it? e.g. via clever security design?

An illustrative example

Two investors, Alice & Bob, participate in funding a risky, scalable project

- independently decide how much money to commit to the project
  – based on their optimal return–risk trade-off
- deep pocketed; identically risk averse

Both investors use their private information to guide investment decisions

- each investor's private information contains idiosyncratic noises
- neither investor has access to the other's private information

Q: How should Alice and Bob divide up any payoff from their investment?

How should Alice and Bob divide up the payoff?

The typical approach (common stock)

- rewards investors in proportion to their initial investment
- the more Alice has invested, the larger payoffs she will enjoy

But...is this really optimal?

- (with common stock) if the project ever goes bust...
- the more Alice has invested, the larger loss she will suffer!

The fear of loss (risk-aversion) limits how much each individual invests

- better risk sharing in compensation may overcome this

$\Rightarrow$ common stock may be suboptimal for harnessing wisdom of the crowd

What if, Alice and Bob **equally** divide up any net payoff?

- i.e. profit sharing!

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Common stock vs. profit sharing

Assume that *net* return is realized as 10% $\uparrow$

Common stock

	Inv.Amt	Shr.G.	Gross payoff	Individual payoff
A	\$200	2/3	$(\$200 + \$100)\times$	$\$330 \times 2/3 = \$220$
B	\$100	1/3	$(1 + 10\%) = \$330$	$\$330 \times 1/3 = \$110$

Fifty-fifty profit sharing (assume no changes in investment)

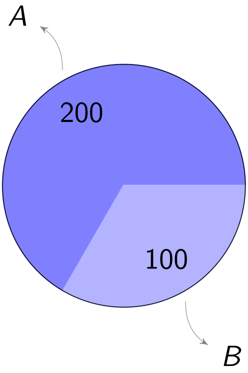
	Shr.N.	Inv.Amt	Net payoff	Individual payoff
A	1/2	\$200	$(\$200 + \$100)\times$	$\$200 + \$30 \times 1/2 = \$215$
B	1/2	\$100	$10\% = \$30$	$\$100 + \$30 \times 1/2 = \$115$

A bad deal for A? Optimal investment also changes under profit sharing...

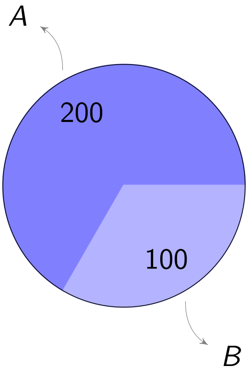
- not necessarily a bad deal to get a smaller piece of a bigger pie!

Common stock vs. profit sharing: illustration

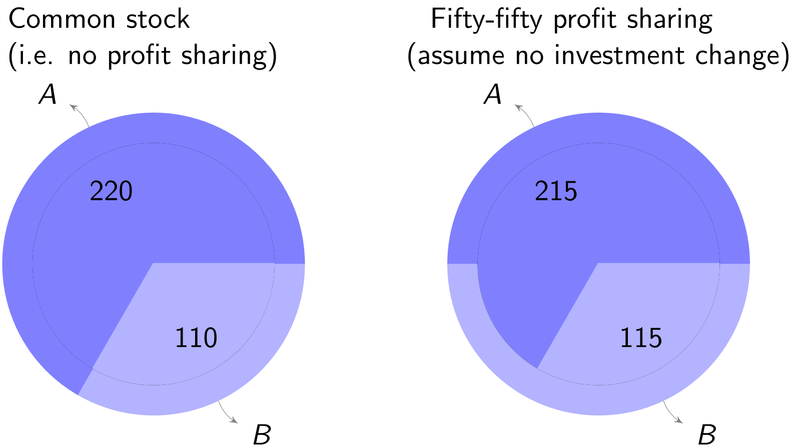
Common stock
(i.e. no profit-sharing)



Fifty-fifty profit sharing
(assume no investment change)



Common stock vs. profit sharing: illustration



Assume that *net* return is realized as 10% $\uparrow$

A bad deal for *A* to go fifty-fifty?

Common stock vs. profit sharing: illustration

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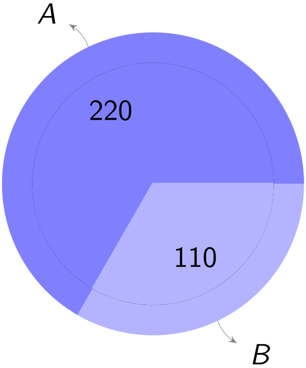
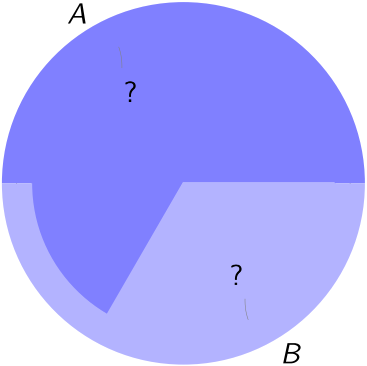


Fifty-fifty profit sharing
(investment optimally changes)



Not necessary...if *A* gets a smaller piece of a bigger pie!

Formal analysis of the illustrative example

Two deep-pocketed, identically risk averse investors ($i \in \{A, B\}$)

- maximize constant absolute risk aversion utility: $u(W) = -e^{-\rho W}$

The risky (scalable) prior with *net* return denoted as a random variable $\tilde{r}$

- investor i's private signal $s_i = r + \epsilon_i$

  where r is the realization of $\tilde{r}$, $\epsilon_i \sim \mathcal{N}(0, \tau_i^{-1})$, $\epsilon_i \perp\!\!\!\perp \tilde{r}$, $\epsilon_A \perp\!\!\!\perp \epsilon_B$

Optimal Investment under common stock

Investor i's problem: invest x_i' given s_i s.t.

$$x_i'(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho \tilde{r} x} | s_i]$$

Assume $\tilde{r} \sim \mathcal{N}(\bar{r}, \tau_r^{-1})$ for ease of exposition, RHS leads to

$$\begin{aligned} x_i'(s_i) &= \text{argmax}_x - e^{-\rho \mathbb{E}(\tilde{r}|s_i)x + \frac{1}{2}\text{Var}(\tilde{r}|s_i)\rho^2 x^2} \\ &= \frac{1}{\rho}(\tau_r \bar{r} + \tau_i s_i) \end{aligned}$$

If A and B could exchange private information before making investing decisions...

Then investor i knew both s_i and s_{-i}, and

$$x_i'(s_i, s_{-i}) = \text{argmax}_x \mathbb{E}[-e^{-\rho\tilde{r}x}|s_A, s_B]$$

RHS leads to

$$\begin{aligned} x_i'(s_A, s_B) &= \text{argmax}_x - e^{-\rho\mathbb{E}(\tilde{r}|s_A,s_B)x+\frac{1}{2}\text{Var}(\tilde{r}|s_A,s_B)\rho^2x^2} \\ &= \frac{1}{\rho}(\tau_r\bar{r} + \tau_A s_A + \tau_B s_B) \end{aligned}$$

(full information benchmark)

Optimal Investment if A and B agree to share profits equally

Investor i's problem: invest x_i given s_i s.t.

$$x_i(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2} \tilde{r}[x + \tilde{x}_{-i}(s_{-i})]} | s_i]$$

$\because$ the RHS involves i's belief of $\tilde{x}_{-i}(s_{-i})$

- solution constitutes a Nash equilibrium

Definition

A Nash Equilibrium under an equal division of profits consists of two investment strategy functions $x_A(\cdot)$ and $x_B(\cdot)$ such that

$$x_i(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2} \tilde{r}[x + \tilde{x}_{-i}(s_{-i})]} | s_i],$$

where $i \in \{A, B\}$ and $-i = \{A, B\} \backslash \{i\}$.

Solving the Nash equilibrium

Nash Equilibrium (from the Definition)

$$x_i(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2} \tilde{r}[x+\tilde{x}_{-i}(s_{-i})]} | s_i], \qquad (1)$$

Guess and verify a linear Nash equilibrium

$$\begin{aligned} x_i(s_i) &= \alpha + \beta_i s_i \\ (1) \Rightarrow \alpha + \beta_i s_i &= \text{argmax}_x - \mathbb{E}[e^{[-\frac{1}{2}\rho\tilde{r}][x+\alpha+\beta_{-i}\tilde{s}_{-i}]} | s_i] \end{aligned} \qquad (2)$$

Both $-\frac{1}{2}\rho\tilde{r}$ and $x + \alpha + \beta_{-i}\tilde{s}_{-i}$ are normal r.v.-s conditional on s_i

$\Rightarrow$ expectation in the RHS of (2): m.g.f of a (general) χ^2-r.v.

- a closed-form expression exists

Profit sharing harnesses crowd wisdom

Under fifty-fifty profit sharing:

$$\begin{cases} x_i &= (\tau_r \bar{r} + 2\tau_i s_i)/\rho \\ x_{-i} &= (\tau_r \bar{r} + 2\tau_{-i} s_{-i})/\rho \end{cases}$$

$$\Rightarrow i\text{'s payoff: } r(x_i + x_{-i})/2 = r(\tau_r \bar{r} + \tau_A s_A + \tau_B s_B)/\rho$$

If A and B exchange private information before investing

$$x_i'(s_i, s_i) = x_{-i}'(s_i, s_i) = (\tau_r \bar{r} + \tau_A s_A + \tau_B s_B)/\rho$$

$$\Rightarrow i\text{'s payoff: } rx_i'(s_i, s_i) = r(\tau_r \bar{r} + \tau_A s_A + \tau_B s_B)/\rho$$

Theorem

$\forall\{r, s_A, s_B\}$, each investor's payoff under an equal division of profits always equals to that under a full information benchmark.

General case: optimal profit-sharing

Consider n investors each with risk-aversion ρ_i and receiving a_i of the profit

Theorem (equilibrium existence and structure)

Iff the pre-agreed profit ratio is proportional to risk tolerance, i.e.

$$a_i = \frac{1/\rho_i}{\sum_{i=1}^{n} 1/\rho_i},$$

a Nash equilibrium exists, under which each investor's payoff is equal to what is under a full information benchmark.

Optimal sharing rule is easy to implement (only requires risk-aversions)

- individuals also have strict incentives to truthfully report their ρ_i-s

Implications for crowdfunding security design

In May 2016, the SEC approved investment crowdfunding

- under Title III of the Jumpstart Our Business Startups (JOBS) Act
- entrepreneurs directly solicit funding from a large number of investors
- contracts agreed to at the time of investment specify monetary payoffs

Q1: What type of contract is optimal? Still an open question.

- currently common stock, debt, or hybrids are all used in practice

Wisdom of the crowd: an acclaimed benefit of crowdfunding

- extensively discussed from the entrepreneur's perspective:
- aggregate investment provides useful information to the entrepreneur

Q2: Could the wisdom of the crowd also benefit investors themselves?

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Thank you!

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and stay tuned for the upcoming two papers!

Investment crowdfunding platforms

nextseed

start engine

AngelList

crowdfunder

Republic

wefunder
crowdfunding for startups and small businesses

How robust is our main result? I

Empirically, only a small number of entrepreneurial ventures take off while most others fail – returns may be skewed...

$\Rightarrow$ our result is intact under skewed project returns

Theorem (Arbitrary distributions of project return)

$\forall$ arbitrary distributions of project return $\tilde{r}$ and an exponential family likelihood function of $\tilde{r}$ given private signals $s_i, i \in \{A, B\}$, profit sharing gives the same payoff for both investors as in a full-information benchmark.

How robust is our main result? II

Sensible to assume endowed private information in crowdfunding

- how will results change if private information has be costly acquired?
- a free-riding problem (Holmström (1982)) in information acquisition?
- e.g. assume constant marginal cost in acquiring signal precision

⇒ free-riding not large enough to cancel out the wisdom of the crowd

Theorem (Costly Information Acquisition)

With a constant marginal cost in acquiring private signal precision, investors strictly prefer more participants in profit sharing.

How robust is our main result? III

Sensible to assume constant return to scale for crowdfunding projects

- how will results change for projects with (dis)economies of scale?
- e.g. assume total investment influences net return $\tilde{r} - \lambda(x_1 + x_2)$

$\Rightarrow$ the profit-sharing contract derived above is still *first-best* optimal

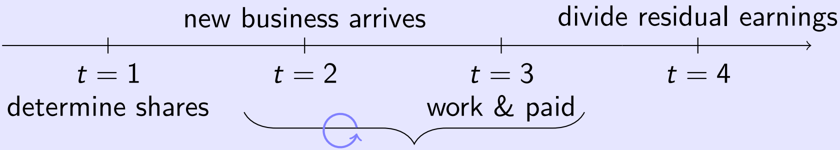
Theorem (Projects with (dis)economy of scale)

The first-best allocation chosen by an omniscient and benevolent social planner could be sustained by a Nash equilibrium under profit sharing plus some cash transfers, even if the project features (dis)economy of scale.

A Second Welfare Theorem under externality and asymmetric information?

A few further thoughts

A partnership firm?



Bitcoin mining pools?

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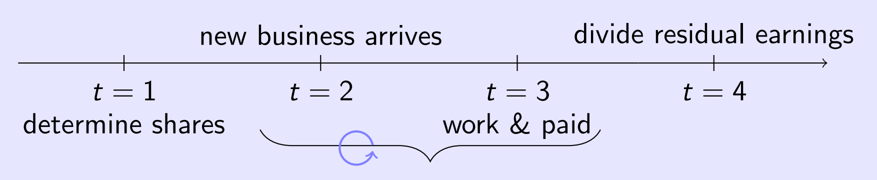

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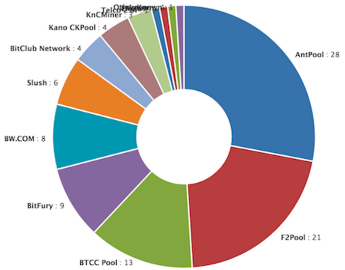

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